IOSR Journal of Electronics and Communication Engineering (IOSR-JECE)
e-ISSN: 2278-2834,p- ISSN: 2278-8735.Volume 10, Issue 2, Ver. III (Mar - Apr.2015), PP 79-82
www.iosrjournals.org

A comparative study on Main and Side lobe of frequency response curve for FIR Filter using Frequency Series Expansion Method

Sourav Chandra[1], Samir Bhowmik[1], Sudipto Bhaumik[1], Subhojit Malik[2]
[1] B.Tech. Student, Electronics and Communication Engineering Department, Hooghly Engineering and Technology College, Hooghly, West Bengal, India
[2] Assistant Professor, Electronics and Communication Engineering Department, Hooghly Engineering and Technology College, Hooghly, West Bengal, India

Abstract: *The spectral contents of a frequency response curve can be modified by the system which is called as filter. To design a filter, the specifications must be used and the calculation of filter coefficients leads to approximate the desired frequency response. When a causal structure and non-recursive in nature with linear phase characteristics is used, the filter will be called as Finite Impulse Response (FIR).The FIR filter can be formed using Fourier Series Expansion Method. The Impulse Response of the filter is designed in discrete time (DT) domain with period equal to 2π considering the response between –π to +π in Fourier Series Expansion Method. So the length of the response is fixed and finite. The value π corresponds to the Nyquist Frequency (F_N) which is equal to $F_S/2$ where F_S is the sampling frequency of the impulse response of the filter. In this paper the response curves for Low Pass, High Pass and Band Pass FIR Filter are observed and compared for different sampling frequency using Fourier Series Expansion Method. The main lobe and side lobe from the characteristics curves are also compared for different types of filters.*

Keywords: *Finite Impulse Response, Fourier Series Expansion, Frequency Response Curve, Main Lobe, Sampling frequency, Side Lobe*

I. Introduction

The impulse response of a digital filter can be two types: Infinite Impulse Response (IIR) and Finite Impulse Response (FIR). In this paper, the design of linear phase FIR filter is discussed using Fourier Series Expansion Method. The frequency response of the digital filter is periodic with period equal to 2π. The periodic function can be expressed as a linear combination of complex exponentials. The desired frequency response $H_d(e^{j\omega})$ can be converted to a Fourier Series representation. Thus the Fourier coefficients can be evaluated to find the desired impulse response $h_d(n)$ of the filter. The transfer function $H_d(z)$ of the digital filter can be obtained by taking Z-transform on $h_d(n)$. The transfer function is not realizable because of the non-causal nature. So a finite duration impulse response h(n) can be obtained by truncating the infinite impulse response $h_d(n)$ to N samples. The duration of the samples is –N to +N. Now H(z) is obtained by taking the Z-transform on h(n) and multiplying H(z) by z^{-N} , the transfer function becomes realizable causal digital filter of finite duration [1-3,5,9].

The design of FIR filters is done using Fourier series method. The characteristics curves are observed for Low Pass Filter (LPF), High Pass Filter (HPF) and Band Pass Filter (BPF). The magnitude values are observed and compared.

In the next section of the paper, the Fourier Series Expansion Method (II), the Finite Impulse Response (III), the Implementation of FIR Filter using Fourier Series Expansion Method (IV), the Conclusions (V) and the References are given.

II. Fourier Series Expansion Method

The Fourier series expansion is used for periodic time domain signals. In times of representing the Fourier series the time domain signal is specified to obtain the frequency response. A digital filter is realized by using a discrete time (DT) system. Hence, the impulse response of the filter to be designed is a DT signal. When the impulse response is converted to frequency domain, it will result in a replicated spectrum with period equal to 2π. The desired response lies between -π to +π and replicate it on either side to get a periodic spectrum with period equal to 2π. Here π corresponds to the Nyquist frequency F_N which is equal to $F_S/2$, where F_S is the sampling frequency of the impulse response of the filter. So the normalized frequency is given by

$$v = \frac{F}{F_N} ----------(1)$$

Now the impulse response can be represented with respect to normalized frequency variable using Fourier series expansion. It is denoted by,

$$H_d(v) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\pi v} ------(2)$$

Now we can observe that,

$$j\omega n = j2\pi fn = \frac{j2\pi Fn}{F_S} = \frac{j2\pi Fn}{2F_N} = jn\pi v ----(3)$$

where F is the analog frequency, f is the frequency of the DT signal corresponds to the impulse response of any system.

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(v)e^{-j\pi nv}dv ------------(4)$$

When the response is plotted within the range of -π to +π, the phase response follows the linearity property and a group delay can be observed as constant which is denoted by

$$\tau = -\frac{d(phase)}{dv} --------------(5)$$

The impulse response with finite duration can be founded by truncating the coefficients between –N to +N. The selection of N should be such that the energy of the coefficients above +N and below – N is very small and is negligible as compared to the energy of the coefficients between –N to +N so that the truncation is justified [4,7-8]. So the modified response can be founded by

$$H_d(v) = \sum_{n=-N}^{+N} C_n e^{jn\pi v} ------------(6)$$

III. Finite Impulse Response

To find the transfer function of the Finite Impulse Response (FIR), we have to find H(z). With H(z), a delay of N samples is introduced to make the response causal and realizable.

$$H(z) = z^{-N} H_d(z) --------------(7)$$

Here, $H_d(z)$ is the transfer function of a non-causal filter with impulse response truncated between –N to +N.
The FIR filter has the following properties:

i) There are 2N+1 number of coefficients of the impulse response.
ii) The impulse response is symmetric with respect to N=0.
iii) The duration of the response is 2NT, where T is the sampling interval=1/Fs.

In times of truncation of impulse response of the filter in between –N to +N, a rectangular window is produced. In frequency domain, this rectangular window produces a sync function. This sync function gives rise to a ripple in the pass band and oscillations in the stop band as well. The oscillations near the band edge of the filter are called Gibbs phenomenon [6,8-10].

IV. Implementation of FIR Filter using Fourier Series Expansion Method

The magnitude response curves are obtained for FIR filter using Fourier series expansion method. The response curves are observed for Low Pass Filter (LPF), High Pass Filter (HPF) and Band Pass Filter (BPF) using the Fourier series method. The sampling frequency is involved for constructing the Fourier series. This sampling frequency (F_S) is varied and naturally the response curves also change. The sampling frequencies which we have taken in this paper are 4000Hz, 8000Hz, 12000Hz, 16000Hz and 20000Hz respectively. To implement and design all these types of FIR filters, MATLAB 7 platform has been used. The maximum magnitude in the main lobe and side lobe are observed and compared for every case. The observations are shown in Table 1 and Table 2 respectively.

V. Result and Simulation

The simulation results for LPF, HPF and BPF with different sampling frequency are shown in the following tables.

Table 1: Table for Comparative study of maximum magnitude of main lobe

Different Sampling Frequency	Maximum Magnitude in Main Lobe (Pass band)		
	LPF	**HPF**	**BPF**
4000	1.0785	1.1150	1.0860
8000	1.0727	1.1041	1.1194
12000	1.1012	1.0810	1.1169
16000	1.0984	1.0824	1.0697
20000	1.0830	1.0953	1.1041

Table 2: Table for Comparative study of maximum magnitude of side lobe

Different Sampling Frequency	Maximum Magnitude in Side Lobe (Stop band)		
	LPF	**HPF**	**BPF**
4000	0.1150	0.0785	0.1595
8000	0.1041	0.0727	0.1225
12000	0.0810	0.1012	0.0775
16000	0.0824	0.0984	0.1180
20000	0.0953	0.0810	0.0862

The graphs show the different magnitude curves for different filters with different sampling frequencies.

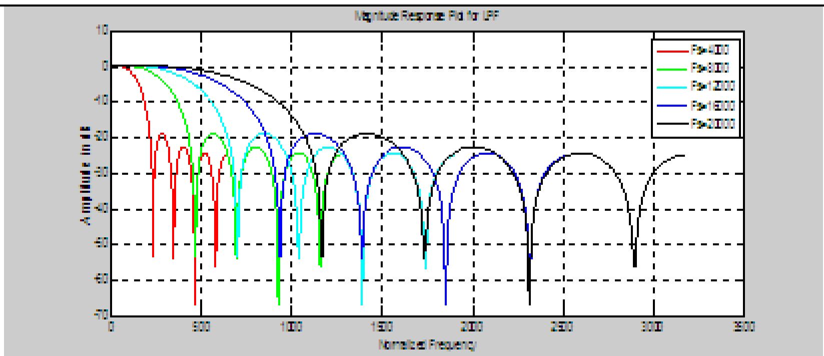
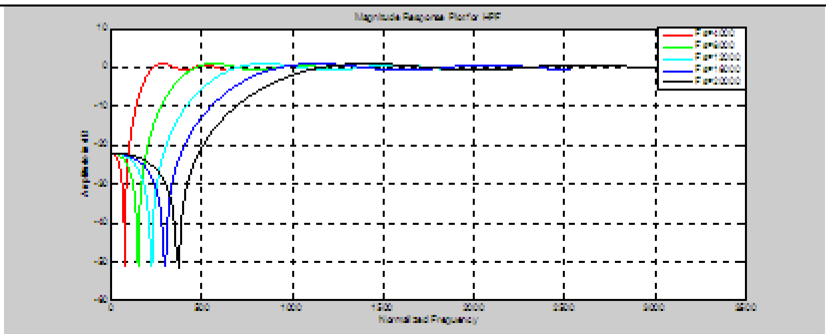
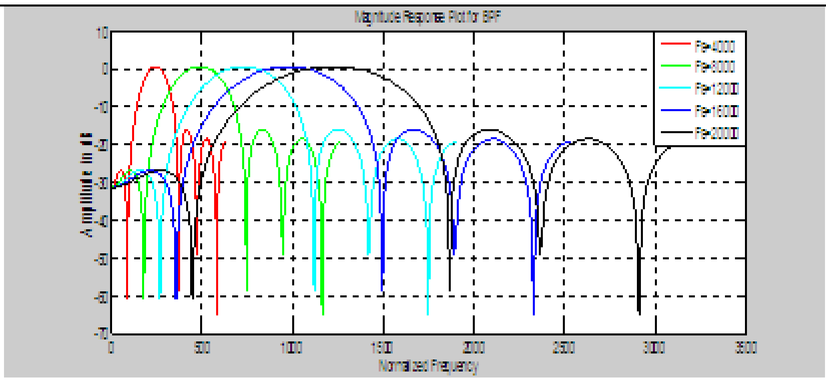
Type of Filter	For Different Sampling Frequencies
LPF	
HPF	
BPF	

Figure: Magnitude Response Curves for FIR Filters for different sampling frequencies

VI. Conclusion

From the different frequency response curves shown in the figure, it is clear that if the sampling frequency is varied, the filter exhibits a deviated response from the ideal response. From comparative study on maximum magnitude of main lobe and side lobe from the frequency response curves, it is quite evident that if the sampling frequency in Fourier series expansion method is increased, the response curves are going towards the ideal response curves with less deviation on magnitudes of different lobes.

Acknowledgements

The authors would like to thank the Department of Electronics and Communication Engineering and the authority of Hooghly Engineering and Technology College, Hooghly, West Bengal, India for providing the facilities and encouraging to carry out research work.

References

[1]. Ben-Dau Tseng, Specifications of ideal prototype filters for designing linear phase FIR filters, 30th Asilomer Conference on Signals, Systems and Computers, 1996

[2]. Kurth R., Design of FIR filters to complex frequency response specifications, IEEE International Conference ICAASSP'82 on Acoustic, Speech, and Processing

[3]. Sahesteh M.G., Mottaghi-Kashtiban M.,An efficient window function for design of FIR filters using IIR FILTERS IEEE Conference EUROCON 2009

[4]. Chen X, Parks T.W., Design of FIR filters in the complex domain, IEEE Transactions on Acoustic, Speech, and Signal Processing, Volume:35, Issue: 2

[5]. Gopal S. Gawande, Dr. K.B.Khanchandani, T.P. Marode, Performance Analysis of FIR digital Filter Design Techniques, International Journal of Computing and Corporate Research, Volume 2, Issue 1, January 2012.

[6]. Chonghua Li, Design and Realization of FIR Digital Filters Based on MATLAB, IEEE 2010.

[7]. Tian-Bo Deng , Yong Lian, Weighted –Least Squares Design of Variable Fractional Delay FIR Filters using Coefficient Symmetry, Signal Processing, IEEE Transaction , 2006

[8]. Dr. Shaila D. Apte, Digital Signal Processing, 2nd Edition, Wiley Publication , 2014

[9]. John G. Proakis, Dimitris G. Manolakis, Digital Signal Processing Principles, Algorithms and Applications, Fourth Edition, PHI Publication, 2006.

[10]. S. Salivahanan, A. Vallavaraj, C. Gnanaapriya, Digital Signal Processing, Tata McGraw-Hill, 2000.